9. $\frac{2(\sec^2 x - \tan^2 x)}{\csc x} = \sin 2x \sec x$

$\frac{2(1)}{\csc x} = \sin 2x \sec x$

$\frac{2}{\csc x} = \sin 2x \sec x$

$2 \sin x = \sin 2x \sec x$

$\frac{2 \sin x \cos x}{\cos x} = \sin 2x \sec x$

$\frac{\sin 2x}{\cos x} = \sin 2x \sec x$

$\sin 2x \sec x = \sin 2x \sec x$

10. a) $x = \frac{7\pi}{6}$ or $\frac{11\pi}{6}$

b) $x = \frac{\pi}{4}$ or $\frac{5\pi}{4}$

c) $x = \frac{2\pi}{3}$ or $\frac{4\pi}{3}$

11. a) $y = -2$ or 2

b) $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}$, or $\frac{11\pi}{6}$

12. a) $x = \frac{\pi}{2}, \frac{7\pi}{6}$, or $\frac{11\pi}{6}$

b) $x = 0, \frac{\pi}{6}, \frac{5\pi}{6}, \pi, \frac{7\pi}{6}, \frac{11\pi}{6}$, or 2π

c) $x = \frac{\pi}{4}, \frac{2\pi}{3}, \frac{4\pi}{3}$, or $\frac{7\pi}{4}$

d) $x = 0.95$ or 4.09

13. $x = \frac{\pi}{2}, \pi$, or $\frac{3\pi}{2}$

Chapter Self-Test, p. 441

1. $\frac{1 - 2\sin^2 x}{\cos x + \sin x} + \sin x = \cos x$

$\frac{1 - 2\sin^2 x}{\cos x + \sin x} + \sin x - \sin x$

$= \cos x - \sin x$

$\frac{1 - 2\sin^2 x}{\cos x + \sin x} = \cos x - \sin x$

$1 - 2\sin^2 x = (\cos x - \sin x)$
$\times (\cos x + \sin x)$

$\cos 2x = (\cos x - \sin x)$
$\times (\cos x + \sin x)$

$\cos 2x = \cos^2 x - \sin^2 x$

$\cos 2x = \cos 2x$

2. all real numbers x, where $0 \le x \le 2\pi$

3. a) $x = \frac{\pi}{6}$ or $x = \frac{11\pi}{6}$

b) $x = \frac{2\pi}{3}$ or $x = \frac{5\pi}{3}$

c) $x = \frac{5\pi}{4}$ or $x = \frac{7\pi}{4}$

4. $a = 2, b = 1$

5. $t = 7, 11, 19$, and 23

6. Nina can find the cosine of $\frac{11\pi}{4}$ by using the formula $\cos(x + y) = \cos x \cos y - \sin x \sin y$. The cosine of π is -1, and the cosine of $\frac{7\pi}{4}$ is $\frac{\sqrt{2}}{2}$. Also, the sine of π is 0, and the sine of $\frac{7\pi}{4}$ is $-\frac{\sqrt{2}}{2}$. Therefore,

$\cos \frac{11\pi}{4} = \cos\left(\pi + \frac{7\pi}{4}\right)$

$= \left(-1 \times \frac{\sqrt{2}}{2}\right) - \left(0 \times -\frac{\sqrt{2}}{2}\right)$

$= -\frac{\sqrt{2}}{2} - 0$

$= -\frac{\sqrt{2}}{2}$

7. $x = 3.31$ or 6.12

8. $-\frac{33}{65}, -\frac{16}{65}$

9. a) $-\frac{4\sqrt{5}}{9}$ c) $\sqrt{\frac{3 - \sqrt{5}}{6}}$

b) $\frac{1}{9}$ d) $\frac{22}{27}$

10. a) $x = -\frac{5\pi}{3}, -\frac{\pi}{3}, \frac{\pi}{3}$, or $\frac{5\pi}{3}$

b) $x = -\frac{4\pi}{3}, -\frac{2\pi}{3}, \frac{2\pi}{3}$, or $\frac{4\pi}{3}$

c) $x = -\pi$ and π

Chapter 8

Getting Started, p. 446

1. a) $\frac{1}{5^2} = \frac{1}{25}$ d) $\sqrt[3]{125} = 5$

b) 1 e) $-\sqrt{121} = -11$

c) $\sqrt{36} = 6$ f) $\left(\sqrt[3]{\frac{27}{8}}\right)^2 = \frac{9}{4}$

2. a) $3^7 = 2187$ d) $7^4 = 2401$

b) $(-2)^2 = 4$ e) $8^{\frac{2}{3}} = 4$

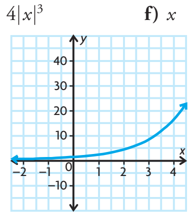
c) $10^3 = 1000$ f) $4^{\frac{1}{2}} = \sqrt{4} = 2$

3. a) $8m^3$ d) x^3y

b) $\frac{1}{a^8b^{10}}$ e) $-d^2c^2$

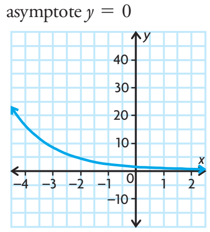
c) $4|x|^3$ f) x

4. a)



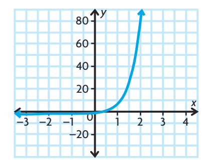
D = {$x \in$ **R**}, R = {$y \in$ **R**$|y > 0$}, y-intercept 1, horizontal asymptote $y = 0$

b)



D = {$x \in$ **R**}, R = {$y \in$ **R**$|y > 0$}, y-intercept 1, horizontal asymptote $y = 0$

c)



D = {$x \in$ **R**}, R = {$y \in$ **R**$|y > -2$}, y-intercept -1, horizontal asymptote $y = -2$

5. a) i) $y = \frac{x + 6}{3}$

ii) $y = \pm\sqrt{x + 5}$

iii) $y = \sqrt[3]{\frac{x}{6}}$

iv)

b) The inverses of (i) and (iii) are functions.

6. a) 800 bacteria

b) 6400 bacteria

c) 209 715 200

d) 4.4×10^{15}

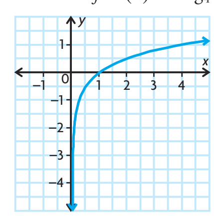
7. 12 515 people

8.

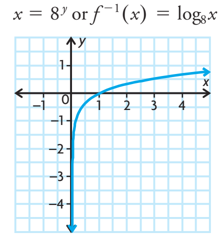
Similarities	Differences
• same y-intercept • same shape • same horizontal asymptote • both are always positive	• one is always increasing, the other is always decreasing • different end behaviour • reflections of each other across the y-axis

Lesson 8.1, p. 451

1. a) $x = 4^y$ or $f^{-1}(x) = \log_4 x$

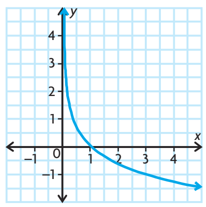


b) $x = 8^y$ or $f^{-1}(x) = \log_8 x$

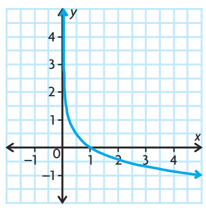


Answers

c) $x = \left(\frac{1}{3}\right)^y$ or $f^{-1}(x) = \log_{\left(\frac{1}{3}\right)}x$



d) $x = \left(\frac{1}{5}\right)^y$ or $f^{-1}(x) = \log_{\left(\frac{1}{5}\right)}x$



2. **a)** **i)** $x = 4^y$
ii) $\log_4 x = y$
b) **i)** $x = 8^y$
ii) $\log_8 x = y$
c) **i)** $x = \left(\frac{1}{3}\right)^y$
ii) $\log_{\frac{1}{3}} x = y$
d) **i)** $x = \left(\frac{1}{5}\right)^y$
ii) $\log_{\frac{1}{5}} x = y$

3. All the graphs have the same basic shape, but the last two are reflected over the x-axis, compared with the first two. All the graphs have the same x-intercept, 1. All have the same vertical asymptote, $x = 0$.

4. Locate the point on the graph that has 8 as its x-coordinate. This point is (8, 3). The y-coordinate of this point is the solution to $2^y = 8$, $y = 3$.

5. **a)** $x = 3^y$ **c)** $x = \left(\frac{1}{4}\right)^y$
b) $x = 10^y$ **d)** $x = m^y$

6. **a)** $\log_3 x = y$ **c)** $\log_{\frac{1}{4}} x = y$
b) $\log_{10} x = y$ **d)** $\log_m x = y$

7. **a)** $x = 5^y$ **c)** $x = 3^y$
b) $x = 10^y$ **d)** $x = \frac{1^y}{4}$

8. **a)** $y = 5^x$ **c)** $y = 3^x$
b) $y = 10^x$ **d)** $y = \frac{1^x}{4}$

9. **a)** 2 **d)** 0
b) 3 **e)** −1
c) 4 **f)** $\frac{1}{2}$

10. Since 3 is positive, no exponent for 3^x can produce −9.

11. **a)** $\left(\frac{1}{4}, -2\right), \left(\frac{1}{2}, -1\right), (1, 0), (2, 1), (4, 2)$
b) $\left(\frac{1}{100}, -2\right), \left(\frac{1}{10}, -1\right), (1, 0), (10, 1), (100, 2)$

Lesson 8.2, pp. 457–458

1. **a)** vertical stretch by a factor of 3
b) horizontal compression by a factor of $\frac{1}{2}$
c) vertical translation 5 units down
d) horizontal translation 4 units left

2. **a)** **(a)** $\left(\frac{1}{10}, -3\right), (1, 0), (10, 3)$
(b) $\left(\frac{1}{20}, -1\right), \left(\frac{1}{2}, 0\right), (5, 1)$
(c) $\left(\frac{1}{10}, -6\right), (1, -5), (10, -4)$
(d) $\left(-3\frac{9}{10}, -1\right), (-3, 0), (6, 1)$
b) **(a)** $D = \{x \in \mathbf{R} \mid x > 0\}$, $R = \{y \in \mathbf{R}\}$
(b) $D = \{x \in \mathbf{R} \mid x > 0\}$, $R = \{y \in \mathbf{R}\}$
(c) $D = \{x \in \mathbf{R} \mid x > 0\}$, $R = \{y \in \mathbf{R}\}$
(d) $D = \{x \in \mathbf{R} \mid x > -4\}$, $R = \{y \in \mathbf{R}\}$

3. **a)** $f(x) = 5 \log_{10} x + 3$
b) $f(x) = -\log_{10}(3x)$
c) $f(x) = \log_{10}(x + 4) - 3$
d) $f(x) = -\log_{10}(x - 4)$

4. **i)** **a)** reflection in the x-axis and a vertical stretch by a factor of 4; $c = 5$ resulting in a translation 5 units up
b) (1, 5), (10, 1)
c) vertical asymptote is $x = 0$
d) $D = \{x \in \mathbf{R} \mid x > 0\}$, $R = \{y \in \mathbf{R}\}$

ii) **a)** vertical compression by a factor of $\frac{1}{2}$; $d = 6$ resulting in a horizontal translation 6 units to the right; $c = 3$ resulting in a vertical translation 3 units up
b) $(7, 3), \left(16, 3\frac{1}{2}\right)$
c) vertical asymptote is $x = 6$
d) $D = \{x \in \mathbf{R} \mid x > 6\}$, $R = \{y \in \mathbf{R}\}$

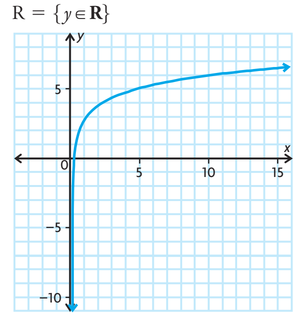
iii) **a)** horizontal compression by a factor of $\frac{1}{3}$; $c = -4$ resulting in a vertical shift 4 units down
b) $\left(\frac{1}{3}, -4\right), \left(3\frac{1}{3}, -3\right)$
c) vertical asymptote is $x = 0$
d) $D = \{x \in \mathbf{R} \mid x > 6\}$, $R = \{y \in \mathbf{R}\}$

iv) **a)** vertical stretch by a factor of 2; $k = -2$ resulting in a horizontal compression by a factor of $\frac{1}{2}$ and a reflection in the y-axis; $d = -2$ resulting in a horizontal translation 2 units to the left.
b) $\left(-2\frac{1}{2}, 0\right), (-7, 2)$
c) vertical asymptote is $x = -2$
d) $D = \{x \in \mathbf{R} \mid x < -2\}$, $R = \{y \in \mathbf{R}\}$

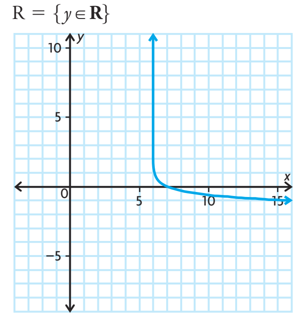
v) **a)** horizontal compression by a factor of $\frac{1}{2}$; $d = -2$ resulting in a horizontal translation 2 units to the left
b) $\left(-1\frac{1}{2}, 0\right), (3, 1)$
c) vertical asymptote is $x = -2$
d) $D = \{x \in \mathbf{R} \mid x > -2\}$, $R = \{y \in \mathbf{R}\}$

vi) **a)** reflection in the x-axis; $d = -2$, resulting in a horizontal translation 2 units to the right
b) $(-3, 0), (-12, 1)$
c) vertical asymptote is $x = -2$
d) $D = \{x \in \mathbf{R} \mid x < -2\}$, $R = \{y \in \mathbf{R}\}$

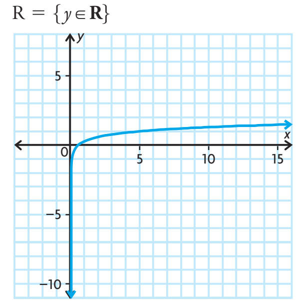
5. **a)** $D = \{x \in \mathbf{R} \mid x > 0\}$, $R = \{y \in \mathbf{R}\}$



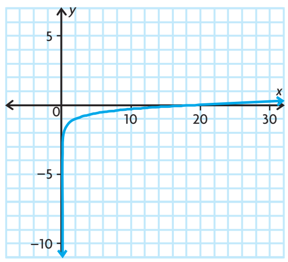
b) $D = \{x \in \mathbf{R} \mid x > -6\}$, $R = \{y \in \mathbf{R}\}$



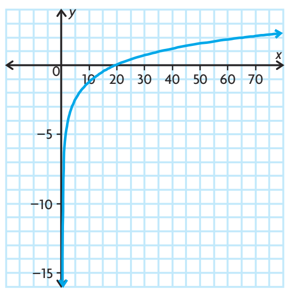
c) $D = \{x \in \mathbf{R} \mid x > 0\}$, $R = \{y \in \mathbf{R}\}$



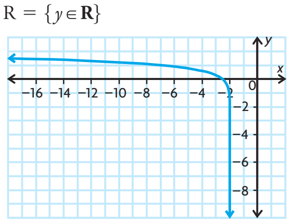
d) D = $\{x \in \mathbf{R} \mid x > 0\}$,
R = $\{y \in \mathbf{R}\}$



e) D = $\{x \in \mathbf{R} \mid x > 0\}$,
R = $\{y \in \mathbf{R}\}$



f) D = $\{x \in \mathbf{R} \mid x < -2\}$,
R = $\{y \in \mathbf{R}\}$



6. The functions are inverses of each other.

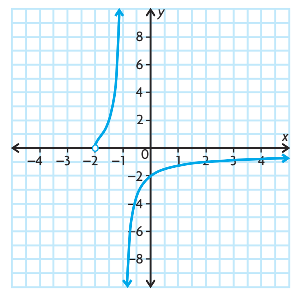
7. **a)** The graph of $g(x) = \log_3(x + 4)$ is the same as the graph of $f(x) = \log_3 x$, but horizontally translated 4 units to the left. The graph of $h(x) = \log_3 x + 4$ is the same as the graph of $f(x) = \log_3 x$, but vertically translated 4 units up.
b) The graph of $m(x) = 4\log_3 x$ is the same as the graph of $f(x) = \log_3 x$, but vertically stretched by a factor of 4. The graph of $n(x) = \log_3 4x$ is the same as the graph of $f(x) = \log_3 x$, but horizontally compressed by a factor of $\frac{1}{4}$.

8. **a)** $f(x) = -3\log_{10}\left(\frac{1}{2}x - 5\right) + 2$
b) $(30, -1)$
c) D = $\{x \in \mathbf{R} \mid x > 5\}$,
R = $\{y \in \mathbf{R}\}$

9. vertical compression by a factor of $\frac{1}{2}$, reflection in the x-axis, horizontal translation 5 units to the left

10. domain, range, and vertical asymptote

11.



Lesson 8.3, pp. 466–468

1. **a)** $\log_4 16 = 2$ **d)** $\log_6 \frac{1}{36} = -2$
b) $\log_3 81 = 4$ **e)** $\log_{\frac{1}{3}} \frac{1}{27} = 3$
c) $\log_8 1 = 0$ **f)** $\log_8 2 = \frac{1}{3}$

2. **a)** $2^3 = 8$ **d)** $\left(\frac{1}{6}\right)^{-3} = 216$
b) $5^{-2} = \frac{1}{25}$ **e)** $6^{\frac{1}{2}} = \sqrt{6}$
c) $3^4 = 81$ **f)** $10^0 = 1$

3. **a)** 1 **d)** $\frac{1}{2}$
b) 0 **e)** 3
c) -2 **f)** $\frac{1}{3}$

4. **a)** -1 **d)** about 25
b) 0 **e)** 1.78
c) 6 **f)** 0.01

5. **a)** $\frac{1}{2}$ **d)** -2
b) 1 **e)** $\frac{1}{3}$
c) 7 **f)** $\frac{3}{2}$

6. **a)** 125 **d)** 16
b) 3 **e)** $\sqrt{5}$
c) -3 **f)** 8

7. **a)** about 2.58 **c)** about 4.29
b) about 3.26 **d)** about 4.52

8. **a)** about 2.50 **c)** about 4.88
b) about 2.65 **d)** about 2.83

9. **a)** 5 **d)** n
b) 25 **e)** b
c) $\frac{1}{16}$ **f)** 0

10. $\frac{4}{3}$

11. about 1.7 weeks or 12 days

12. **a)** 4.68 g **b)** 522 years

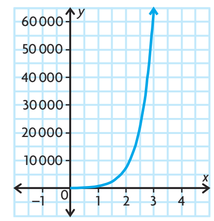
13. A: $(0.0625) = 0.017$; B: $(1) = 0.159$; B has a steeper slope.

14. **a)** about 233 mph **b)** 98 miles

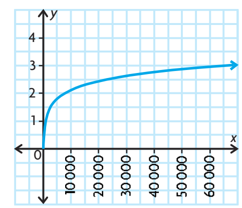
15. $\log 365 = 2.562$
$\frac{3}{2}\log 150 - 0.7 = 2.564$

16. **a)** about 83 years
b) about 164 years

17. **a)** $y = 100(2)^{\frac{x}{0.32}}$
b)



c)



d) $y = 0.32\log_2\left(\frac{x}{100}\right)$; this equation tells how many hours, y, it will take for the number of bacteria to reach x.
e) about 0.69 h; evaluate the inverse function for $x = 450$

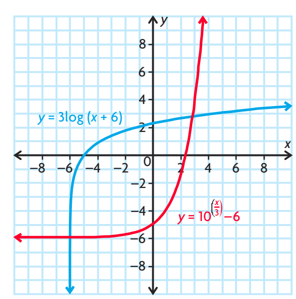
18. **a)** 1.0000 **d)** 2.1745
b) 3.3219 **e)** -0.5000
c) 2.3652 **f)** 2.9723

19. **a)** positive for all values $x > 1$
b) negative for all values $0 < x < 1$
c) undefined for all values $x \leq 0$

20. **a)** 1027
b) -27.14

21. **a)** $y = x^3$ **c)** $\sqrt[(x-2)]{0.5}$
b) $\frac{\sqrt[4]{2}}{3}$ **d)** $2^{\frac{x-2}{3}} + 3$

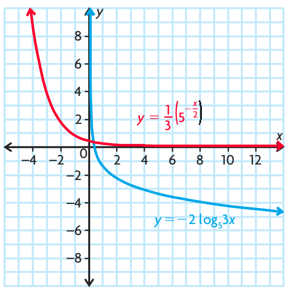
22. **a)**



function: $y = 3\log(x + 6)$
D = $\{x \in \mathbf{R} \mid x > -6\}$
R = $\{y \in \mathbf{R}\}$
asymptote: $x = -6$

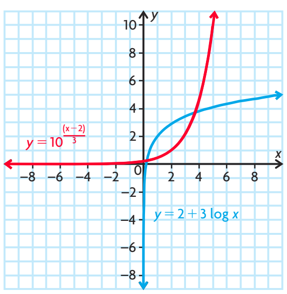
inverse: $y = 10^{\frac{x}{3}} - 6$
D = $\{x \in \mathbf{R}\}$
R = $\{y \in \mathbf{R} \mid y > -6\}$
asymptote: $y = -6$

b)



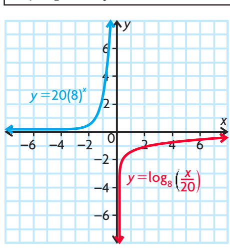
function: $y = -2\log_5 3x$ $D = \{x \in \mathbf{R} \mid x > 0\}$ $R = \{y \in \mathbf{R}\}$ asymptote: $x = 0$
inverse: $y = \frac{1}{3}\left(5^{-\frac{x}{2}}\right)$ $D = \{x \in \mathbf{R}\}$ $R = \{y \in \mathbf{R} \mid y > 0\}$ asymptote: $y = 0$

c)



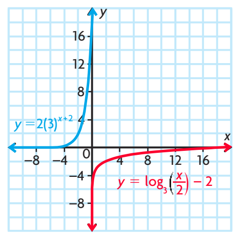
function: $y = 2 + 3\log x$ $D = \{x \in \mathbf{R} \mid x > 0\}$ $R = \{y \in \mathbf{R}\}$ asymptote: $x = 0$
inverse: $y = 10^{\frac{(x-2)}{3}}$ $D = \{x \in \mathbf{R}\}$ $R = \{y \in \mathbf{R} \mid y > 0\}$ asymptote: $y = 0$

d)



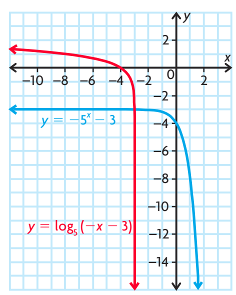
function: $y = 20(8)^x$ $D = \{x \in \mathbf{R}\}$ $R = \{y \in \mathbf{R} \mid y > 0\}$ asymptote: $y = 0$
inverse: $y = \log_8\left(\frac{x}{20}\right)$ $D = \{x \in \mathbf{R} \mid x > 0\}$ $R = \{y \in \mathbf{R}\}$ asymptote: $x = 0$

e)



function: $y = 2(3)^{x+2}$ $D = \{x \in \mathbf{R}\}$ $R = \{y \in \mathbf{R} \mid y > 0\}$ asymptote: $y = 0$
inverse: $y = \log_3\left(\frac{x}{2}\right) - 2$ $D = \{x \in \mathbf{R} \mid x > 0\}$ $R = \{y \in \mathbf{R}\}$ asymptote: $x = 0$

f)



function: $y = -5^x - 3$ $D = \{x \in \mathbf{R}\}$ $R = \{y \in \mathbf{R} \mid y < -3\}$ asymptote: $y = -3$
inverse: $y = \log_5(-x-3)$ $D = \{x \in \mathbf{R} \mid x < -3\}$ $R = \{y \in \mathbf{R}\}$ asymptote: $x = -3$

23. Given the constraints, two integer values are possible for y, either 1 or 2. If $y = 3$, then x must be 1000, which is not permitted.

Lesson 8.4, pp. 475–476

1. **a)** $\log 45 + \log 68$
b) $\log_m p + \log_m q$
c) $\log 123 - \log 31$
d) $\log_m p - \log_m q$
e) $\log_2 14 + \log_2 9$
f) $\log_4 81 - \log_4 30$

2. **a)** $\log 35$ **d)** $\log \frac{x}{y}$
b) $\log_3 2$ **e)** $\log_6 504$
c) $\log_m ab$ **f)** $\log_4 6$

3. **a)** $2\log 5$ **d)** $\frac{1}{3}\log 45$
b) $-1\log 7$ **e)** $\frac{1}{2}\log_7 36$
c) $q\log_m p$ **f)** $\frac{1}{5}\log_5 125$

4. **a)** $\log_3 27$; 3 **d)** $7\log_4 4$; 7
b) $\log_5 25$; 2 **e)** $\log_2 32$; 5
c) $\log 100$; 2 **f)** $\frac{1}{2}\log 10$; $\frac{1}{2}$

5. $y = \log_2(4x) = \log_2 x + \log_2 4$
$= \log_2 x + 2$, so $y = \log_2(4x)$ vertically shifts $y = \log_2 x$ up 2 units;
$y = \log_2(8x) = \log_2 x + \log_2 8$
$= \log_2 x + 3$, so $y = \log_2(4x)$ vertically shifts $y = \log_2 x$ up 3 units;
$y = \log_2\left(\frac{x}{2}\right) = \log_2 x - \log_2 2$
$= \log_2 x - 1$, so $y = \log_2(4x)$ vertically shifts $y = \log_2 x$ down 1 unit

6. **a)** 1.5 **d)** -0.5
b) 2 **e)** 4
c) 1.5 **f)** 2

7. **a)** $\log_b x + \log_b y + \log_b z$
b) $\log_b z - (\log_b x + \log_b y)$
c) $2\log_b x + 3\log_b y$
d) $\frac{1}{2}(5\log_b x + \log_b y + 3\log_b z)$

8. $\log_5 3$ means $5^x = 3$ and $\log_5 \frac{1}{3}$ means $5^y = \frac{1}{3}$; since $\frac{1}{3} = 3^{-1}$, $5^y = 5^{x(-1)}$; therefore $\log_5 3 + \log_5 \frac{1}{3} = x + x(-1) = 0$

9. **a)** $\log_5 56$ **d)** $\log_3 4$
b) $\log_3 2$ **e)** $\log_4(3\sqrt{2})$
c) $\log_2 45$ **f)** $\log 16$

10. **a)** $\log_2 x = \log_2 245$; $x = 245$
b) $\log x = \log 432$; $x = 432$
c) $\log_4 x = \log_7 5$; $x = 5$
d) $\log_7 x = \log_7 5$; $x = 5$
e) $\log_3 x = \log_3 4$; $x = 4$
f) $\log_5 x = \log_5 384$; $x = 384$

11. **a)** $\log_2 xyz$ **d)** $\log_2 xy$
b) $\log_5 \frac{uw}{v}$ **e)** $\log_3 3x^2$
c) $\log_6 \frac{a}{bc}$ **f)** $\log_4 \frac{x^5}{v}$

12. $\log_a \frac{\sqrt{x}\sqrt{y}}{\sqrt[4]{z^3}}$

13. vertical stretch by a factor of 3, and vertical shift 3 units up

14. Answers may vary. For example,
$f(x) = 2\log x - \log 12$
$g(x) = \log \frac{x^2}{12}$
$2\log x - \log 12 = \log x^2 - \log 12$
$= \log \frac{x^2}{12}$

15. Answers may vary. For example, any number can be written as a power with a given base. The base of the logarithm is 3. Write each term in the quotient as a power of 3. The laws of logarithms make it possible to evaluate the expression by simplifying the quotient and noting the exponent.

16. $\log_x x^{m-1} + 1 = m - 1 + 1 = m$

17. $\log_b x\sqrt{x} = \log_b x + \log_b \sqrt{x}$
$= \log_b x + \frac{1}{2}\log_b x$
$= 0.3 + 0.3\left(\frac{1}{2}\right)$
$= 0.45$

18. The two functions have different domains. The first function has a domain of $x > 0$. The second function has a domain of all real numbers except 0, since x is squared.

19. Answers may vary; for example,
Product law
$\log_{10}10 + \log_{10}10 = 1 + 1$
$= 2$
$= \log_{10}100$
$= \log_{10}(10 \times 10)$
Quotient law
$\log_{10}10 - \log_{10}10 = 1 - 1$
$= 0$
$= \log_{10}1$
$= \log_{10}\left(\frac{10}{10}\right)$
Power law
$\log_{10}10^2 = \log_{10}100$
$= 2$
$= 2\log_{10}10$

Mid-Chapter Review, p. 479

1. **a)** $\log_5 y = x$ **b)** $\log_{\frac{1}{3}} y = x$ **c)** $\log x = y$ **d)** $\log_p m = q$

2. **a)** $3^y = x$ **b)** $10^y = x$ **c)** $10^k = m$ **d)** $s^t = r$

3. **a)** vertical stretch by a factor of 2, vertical translation 4 units down
b) reflection in the x-axis, horizontal compression by a factor of $\frac{1}{3}$
c) vertical compression by a factor of $\frac{1}{4}$, horizontal stretch by a factor of 4
d) horizontal compression by a factor of $\frac{1}{2}$, horizontal translation 2 units to the right
e) horizontal translation 5 units to the left, vertical translation 1 unit up
f) vertical stretch by a factor of 5, reflection in the y-axis, vertical translation 3 units down

4. **a)** $y = -4\log_3 x$
b) $y = \log_3(x + 3) + 1$
c) $y = \frac{2}{3}\log_3\left(\frac{1}{2}x\right)$
d) $y = 3\log_3[-(x - 1)]$

5. **a)** $(9, -8)$
b) $(6, 3)$
c) $\left(18, \frac{4}{3}\right)$
d) $(-8, 6)$

6. It is vertically stretched by a factor of 2 and vertically shifted up 2.

7. **a)** 4 **b)** -2 **c)** 0 **d)** -3

8. **a)** 0.602 **b)** 1.653 **c)** 2.130 **d)** 2.477

9. **a)** $x \doteq 4.392$ **b)** $x \doteq 2.959$ **c)** $x \doteq 2.543$ **d)** $x \doteq 2.450$

10. **a)** $\log 28$ **b)** $\log 2.5$ **c)** $\log_3\frac{22}{3}$ **d)** $\log_p q^2$

11. **a)** 1 **b)** 2 **c)** 2 **d)** -3 **e)** $\frac{2}{3}$ **f)** 3.5

12. Compared with the graph of $y = \log x$, the graph of $y = \log x^3$ is vertically stretched by a factor of 3.

13. **a)** 4.82 **b)** 1.35 **c)** 0.80 **d)** 1.69 **e)** 3.82 **f)** 3.49

Lesson 8.5, pp. 485–486

1. **a)** 4 **b)** 1 **c)** $\frac{11}{4}$ **d)** $\frac{13}{9}$ **e)** $-\frac{1}{3}$ **f)** -1

2. **a)** 4.088 **b)** 3.037 **c)** 1 **d)** 4.092 **e)** -0.431 **f)** 5.695

3. **a)** 5 **b)** 3 **c)** 1.5 **d)** $\frac{3}{5}$ **e)** -2 **f)** $-\frac{1}{2}$

4. **a)** 4.68 h **b)** 12.68 h **c)** 16 h **d)** 31.26 h

5. **a)** 1.75 **b)** $\frac{2}{3}$ **c)** -4.75 **d)** -4 **e)** 2 **f)** 2

6. **a)** 9.12 years
b) 13.5 years
c) 16.44 quarters or 4.1 years
d) 477.9 weeks or 9.2 years

7. 13 quarter hours or 3.25 h

8. **a)** 2.5 **b)** 6 **c)** 5 **d)** 3 **e)** 1 **f)** 0

9. **a)** Solve using logarithms. Both sides can be divided by 225, leaving only a term with a variable in the exponent on the left. This can be solved using logarithms.
b) Solve by factoring out a power of 3 and then simplifying. Logarithms may still be necessary in a situation like this, but the factoring must be done first because logarithms cannot be used on the equation in its current form.

10. **a)** 1.849 **b)** 2.931 **c)** 3.606 **d)** 5.734

11. **a)** $I_f = I_o(0.95)^t$, where I_f is the final intensity, I_o is the original intensity, and t is the thickness
b) 10 mm

12. 1; 0.631

13. $a^y = x$, so $\log a^y = \log x$; $y\log a = \log x$;
$y = \frac{\log x}{\log a}$
A graphing calculator does not allow logarithms of base 5 to be entered directly. However, $y = \log_5 x$ can be entered for graphing, as $y = \frac{\log x}{\log 5}$.

14. **a)** $x = 2.5$
b) $x = 5$ or $x = 4$
c) $x = -2.45$

15. Let $\log_a 2 = x$. Then $a^x = 2$. $(a^x)^3 = 2^3$, or $a^{3x} = 8$. Since $\log_a 2 = \log_b 8$, $\log_b 8 = x$. So $b^x = 8$. Since each equation is equal to 8, $a^{3x} = b^x$ and $a^3 = b$.

16. $x = -0.737$; $y = 0.279$

17. **a)** $x = -1.60$
b) $x = -4.86$
c) $x = -0.42$

18. ± 1.82

Lesson 8.6, pp. 491–492

1. **a)** 25 **b)** 81 **c)** 8 **d)** 15 **e)** 3 **f)** $\sqrt{3}$

2. **a)** 5 **b)** $\frac{1}{36}$ **c)** 13 **d)** 200.4 **e)** 5 **f)** 20

3. 201.43

4. **a)** 9 **b)** $\sqrt{5}$ **c)** $\frac{25}{3}$ **d)** 10 000 **e)** -3 **f)** 4

5. **a)** $\frac{8}{3}$ **b)** $\frac{10}{3}$ **c)** $\frac{25}{6}$ **d)** 32 **e)** 3 **f)** 8.1

6. $x = 9$ or $x = -4$
Restrictions: $x > 5$ ($x - 5$ must be positive) so $x = 9$

7. **a)** $x = 6$ **b)** $x = 3$ **c)** $x = \frac{6}{5}$ **d)** $x = 2.5$ **e)** $x = 3$ **f)** $x = 16$

8. **a)** Use the rules of logarithms to obtain $\log_9 20 = \log_9 x$. Then, because both sides of the equation have the same base, $20 = x$.
b) Use the rules of logarithms to obtain $\log\frac{x}{2} = 3$. Then use the definition of a logarithm to obtain $10^3 = \frac{x}{2}$; $1000 = \frac{x}{2}$; $2000 = x$.

c) Use the rules of logarithms to obtain $\log x = \log 64$. Then, because both sides of the equation have the same base, $x = 64$.

9. a) 10^{-7}
b) $10^{-3.6}$

10. $x = 2.5$ or $x = 2$

11. a) $x = 0.80$
b) $x = -6.91$
c) $x = 3.16$
d) $x = 0.34$

12. $x = 4.83$

13. $\log_3(-8) = x$; $3^x = -8$; Raising positive 3 to any power produces a positive value. If $x \geq 1$, then $3^x \geq 3$. If $0 \leq x < 1$, then $1 \leq x < 3$. If $x < 0$, then $0 < x < 1$.

14. a) $x > 3$
b) If x is 3, we are trying to take the logarithm of 0. If x is less than 3, we are trying to take the logarithm of a negative number.

15. $\frac{1}{2}(\log x + \log y) = \frac{1}{2}\log xy = \log\sqrt{xy}$ so $\frac{x+y}{5} = \sqrt{xy}$ and $x + y = 5\sqrt{xy}$. Squaring both sides gives $(x+y)^2 = 25xy$. Expanding gives $x^2 + 2xy + y = 25xy$; therefore, $x + y = 23xy$.

16. $x = 3$ or $x = 2$

17. 1 and 16, 2 and 8, 4 and 4, 8 and 2, and 16 and 1

18. $x = 4$, $y = 4.58$

19. a) $x = 3$
b) $x = 16$

20. $x = -1.75$, $y = -2.25$

Lesson 8.7, pp. 499–501

1. First earthquake: $5.2 = \log x$; $10^{5.2} = 158\,489$
Second earthquake; $6 = \log x$; $10^6 = 1\,000\,000$
Second earthquake is 6.3 times stronger than the first.

2. 7.2

3. 60 dB

4. 7.9 times

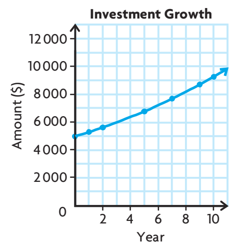
5. a) 0.000 000 001
b) 0.000 000 251
c) 0.000 000 016
d) 0.000 000 000 000 1

6. a) 3.49
b) 3.52
c) 4.35
d) 2.30

7. a) 7
b) Tap water is more acidic than distilled water as it has a lower pH than distilled water (pH 7).

8. 7.98 times

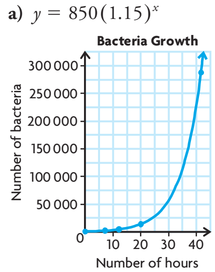
9. a) $y = 5000(1.0642)^t$



b) 6.42%
c) 11.14 years

10. 2.90 m

11. a) $y = 850(1.15)^x$



b) 4.9 h

12. a) 1.22, 1.43, 1.69, 2.00, 2.18, 2.35
b) 1.81
c) $w = 5.061\,88(1.061\,8)^t$
d) $w = 5.061\,88(1.061\,8)^t$
e) 11.5 °C

13. 33 cycles

14. 7.4 years

15. 26.2 days

16. Answers may vary. For example: (1) Tom invested $2000 in an account that accrued interest, compounded annually, at a rate of 6%. How long will it take for Tom's investment to triple? (2) Indira invested $5000 in a stock that made her $75 every month. How long will it take her investment to triple?
The first problem could be modelled using an exponential function. Solving this problem would require the use of logarithms. The second problem could be modelled using a linear equation. Solving the second problem would not require the use of logarithms.

17. 73 dB

18. a) $C = P(1.038)^t$
b) $580.80
c) $33.07

Lesson 8.8, pp. 507–508

1. a) -7.375
b) -23.25
c) -2

2. The instantaneous rate of decline was greatest in year 1. The negative change from year 1 to year 2 was 50, which is greater than the negative change in any other two-year period.

3. a) -12.378
b) -4.867
c) -1.914

4. a) $A(t) = 6000(1.075)^t$
b) 894.35
c) 461.25

5. a) i) 61.80
ii) 67.65
iii) 79.08
b) The rate of change is not constant because the value of the account each year is determined by adding a percent of the previous year's value.

6. a) 20.40 g
b) -0.111 g/h

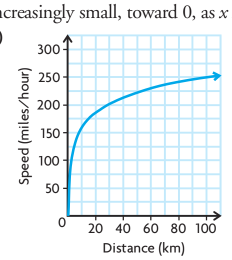
7. a) 1.59 g/day
b) $y = 0.0017(1.7698)^x$, where x is the number of days after the egg is laid
c) i) 0.0095 g/day
ii) 0.917 g/day
iii) 88.25 g/day
d) 14.3 days

8. a) 3.81 years
b) 9.5%/year

9. a) $y = 12\,000(0.982)^t$
b) -181.7 people/year
c) -109 people/year

10. Both functions approach a horizontal asymptote. Each change in x yields a smaller and smaller change in y. Therefore, the instantaneous rate of change grows increasingly small, toward 0, as x increases.

11. a)



b) 1.03 miles/hour/hour
c) 4.03 miles/hour/hour and 0.403 miles/hour/hour
d) The rate at which the wind changes during shorter distances is much greater than the rate at which the wind changes at farther distances. As the distance increases, the rate of change approaches 0.

12. To calculate the instantaneous rate of change for a given point, use the exponential function to calculate the values of y that approach the given value of x. Do this for values on either side of the given value of x. Determine the average rate of change for these values of x and y. When the average rate of change has stabilized to a constant value, this is the instantaneous rate of change.

13. **a)** and **b)** Only a and k affect the instantaneous rate of change. Increases in the absolute value of either parameter tend to increase the instantaneous rate of change.

Chapter Review, pp. 510–511

1. **a)** $y = \log_4 x$ **c)** $y = \log_{\frac{3}{4}} x$
b) $y = \log_a x$ **d)** $m = \log_p q$

2. **a)** vertical stretch by a factor of 3, reflection in the x-axis, horizontal compression by a factor of $\frac{1}{2}$
b) horizontal translation 5 units to the right, vertical translation 2 units up
c) vertical compression by a factor of $\frac{1}{2}$, horizontal compression by a factor of $\frac{1}{5}$
d) horizontal stretch by a factor of 3, reflection in the y-axis, vertical shift 3 units down

3. **a)** $y = \frac{2}{5} \log x - 3$
b) $y = -\log\left[\frac{1}{2}(x - 3)\right]$
c) $y = 5 \log(-2x)$
d) $y = \log(-x - 4) - 2$

4. Compared to $y = \log x$, $y = 3 \log(x - 1) + 2$ is vertically stretched by a factor of 3, horizontally translated 1 unit to the right, and vertically translated 2 units up.

5. **a)** 3 **c)** 0
b) -2 **d)** -4

6. **a)** 3.615 **c)** 2.829
b) -1.661 **d)** 2.690

7. **a)** log 55 **c)** $\log_5 4$
b) log 5 **d)** log 128

8. **a)** 1 **c)** $\frac{2}{3}$
b) 2 **d)** 3

9. It is shifted 4 units up.

10. **a)** 5 **c)** -2
b) 3.75 **d)** -0.2

11. **a)** 2.432 **c)** 2.553
b) 3.237 **d)** 4.799

12. **a)** 0.79; 0.5
b) -0.43

13. 5.45 days

14. **a)** 63 **c)** 9
b) $\frac{10\,000}{3}$ **d)** 1.5

15. **a)** 1 **c)** 3
b) 5 **d)** $\pm\sqrt{10\,001}$

16. 10^{-2} W/m^2

17. $10^{-3.8}$ W/m^2

18. 5 times

19. 3.9 times

20. $\frac{10^{4.7}}{10^{2.3}} = 251.2$
$\frac{10^{12.5}}{10^{10.1}} = 251.2$
The relative change in each case is the same. Each change produces a solution with concentration 251.2 times the orignial solution.

21. Yes; $y = 3(2.25^x)$

22. 17.8 years

23. **a)** 8671 people per year
b) 7114; The rate of growth for the first 30 years is slower than the rate of growth for the entire period.
c) $y = 134\,322(1.03^x)$, where x is the number of years after 1950
d) **i)** 7171 people per year
ii) 12 950 people per year

24. **a)** exponential; $y = 23(1.17^x)$, where x is the number of years since 1998
b) 331 808
c) Answers may vary. For example, I assumed that the rate of growth would be the same through 2015. This is not reasonable. As more people buy the players, there will be fewer people remaining to buy them, or newer technology may replace them.
d) about 5300 DVD players per year
e) about 4950 DVD players per year
f) Answers may vary. For example, the prediction in part e) makes sense because the prediction is for a year covered by the data given. The prediction made in part b) does not make sense because the prediction is for a year that is beyond the data given, and conditions may change, making the model invalid.

Chapter Self-Test, p. 512

1. **a)** $x = 4^y$; $\log_4 x = y$
b) $y = 6^x$; $\log_6 y = x$

2. **a)** horizontal compression by a factor of $\frac{1}{2}$, horizontal translation 4 units to the right, vertical translation 3 units up
b) vertical compression by a factor of $\frac{1}{2}$, reflection in the x-axis, horizontal translation 5 units to the left, vertical translation 1 unit down

3. **a)** -2 **b)** 5

4. **a)** 2 **b)** 7

5. $\log_4 xy$

6. 7.85

7. **a)** 2 **b)** $1\frac{3}{4}$

8. **a)** 50 g
b) $A(t) = 100(0.5)^{\frac{t}{5730}}$
c) 1844 years
d) -0.015 g/year

9. **a)** 6 min
b) 97°

Chapter 9

Getting Started, p. 516

1. **a)** $f(-1) = 30$, $f(4) = 0$
b) $f(-1) = -2$, $f(4) = -5\frac{1}{3}$
c) $f(-1)$ is undefined, $f(4) \doteq 1.81$
d) $f(-1) = -20$, $f(4) = -0.625$

2. $D = \{x \in \mathbf{R} \mid x \neq 1\}$
$R = \{y \in \mathbf{R} \mid y \neq 2\}$
There is no minimum or maximum value;
the function is never increasing;
the function is decreasing from $(-\infty, 1)$ and $(1, \infty)$;
the function approaches $-\infty$ as x approaches 1 from the left and ∞ as x approaches 1 from the right;
vertical asymptote is $x = 1$;
horizontal asymptote is $y = 2$

3. **a)** $y = 2|x - 3|$
b) $y = -\cos(2x)$
c) $y = \log_3(-x - 4) - 1$
d) $y = -\frac{4}{x} - 5$

4. **a)** $x = -1, \frac{1}{2}$, and 4
b) $x = -\frac{5}{3}$ or $x = 3$
c) $x = 5$ or $x = -2$
Cannot take the log of a negative number, so $x = 5$.
d) $x = -\frac{3}{4}$
e) $x = -3$
f) $\sin x = \frac{3}{2}$ or $\sin x = -1$. Since $\sin x$ cannot be greater than 1, the first equation does not give a solution; $x = 270°$

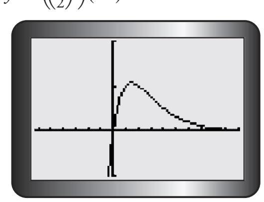
5. **a)** $(-\infty, -4) \cup (2, 3)$
b) $\left(-2, \frac{3}{2}\right) \cup [4, \infty)$

6. **a)** odd **c)** even
b) neither **d)** neither

7. Polynomial, logarithmic, and exponential functions are continuous. Rational and trigonometric functions are sometimes continuous and sometimes not.

Lesson 9.1, p. 520

1. Answers may vary. For example, the graph of $y = \left(\left(\frac{1}{2}\right)^x\right)(2x)$ is



Answers